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Review of Wavelet Transform and KLT

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***Abstract* – This paper contains the review of WT and KLT. Wavelet Transform and Karhunen-Loeve Transform (KLT) are used for image enhancement. The WT is known for its denoise ability and KLT for de-correlation ability.**

Index Terms – WT (Wavelet transform), KLT (Karhunen-Loeve transform), Discrete Fourier Transform (DFT), Discrete Fourier Transform (DFT).

1. INTRODUCTION

Digital images important in daily life applications. Wavelet transform and KLT have been used for image denoising. There are many ways to denoise an image or a set of data. The main properties of a good image denoising model is that it will remove noise while preserving edges. Traditionally, Wavelet transform and KLT have been used. One common approach is to use a Wavelet transform with the noisy image as input-data.

2. WAVELET TRANSFORM

Several transforms are available like Fourier transform, Hilbert transform, Wavelet transform, etc. The wavelet transform is better than other transforms because of following reasons:

- Wavelet transform is better than fourier transform because it gives frequency representation of raw signal at any given interval of time, but fourier transform gives only the frequency- amplitude representation of the raw signal, but the time information is lost. So we cannot use the Fourier transform where we need time as well as frequency information at the same time.
- Wavelet transform is better than discrete fourier transform (DFT) as wavelet transform can capture the localized feature which is the frequency spectrum of a small time segment and while the DFT only offer the global feature that will analysis the global frequency spectrum. Second point is that the computation time which is N for the WT and NlogN for the DFT. The third dissimilarity will be given here is that the difference in the energy distribution, with the coefficient ascend the energy percentage of the discrete wavelet transform (DWT) descend slightly while the discrete fourier transform (DFT) descend firstly then increases dramatically.
- Wavelet transform is better than fast fourier transform (FFT) as individual wavelet functions are localized in time. Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.
- Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. It has an oscillating wavelike characteristic & it as time-scale and time-frequency analysis tools have been widely used in reconstruction and still growing. In wavelet analysis, the scale that used to look at data plays an important role. Wavelet algorithms process data at different scales or resolutions. Looking at a signal with a large "window," that would notice global features. Similarly, looking at a signal with a small "window," that would notice localized features.
- The wavelet transform (WT) is a powerful tool of signal processing for its multiresolutional possibilities. The wavelet transform is suitable for application to non-stationary signals with transitory phenomena, where frequency response varies in time. The wavelet coefficient represents a measure of similarity in the frequency content between a signal and a chosen wavelet function. These coefficient are computed as a convolution of the signal and the scaled wavelet function, which can be interpreted as a dilated band pass filter because of its band pass like spectrum. The discrete wavelet transform (DWT) requires less space utilizing the space saving coding based on the fact that wavelet families are orthogonal or biorthogonal bases, and thus do not produce redundant analysis. The discrete wavelet transform corresponds to its continuous version sampled usually on a dyadic grid, which means that the scales and

translations are power of two. Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing against threshold. If the coefficient is smaller than threshold then it set to be zero; otherwise it is kept or modified. We replace the small noisy coefficient by zero and inverse wavelet transform on the result may lead to reconstruction with the essential signal characteristics and with less noise.

2.1. Continuous Wavelet Transform (CWT)

In definition, the continuous wavelet transform is a convolution of the input data sequence with the set of functions which are generated by the mother wavelet. The convolution is computed by using the Fast Fourier Transform (FFT).Normally, the output is a real function except in the condition when the mother wavelet is complex. A complex mother wavelet will convert the continuous wavelet transform to a complex valued function.

The continuous wavelet transform (CWT) of a signal f(x) is defined as:

$$\mathcal{W}f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x)\psi^*\left(\frac{x-b}{a}\right)dx \ = \ \langle f, \psi_{a,b} \rangle,$$

Where $\psi^*(x)$ denotes the complex conjugate of $\psi(x)$. The existence of the inverse transform is guaranteed if

$$\int_{-\infty}^{\infty} |\hat{\psi}(\omega)|^2/|\omega| d\omega \ \triangleq \ C_\psi \ < +\infty$$

Where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(x)$. This is called the admissibility condition. If $\psi(x)$ can be viewed as an impulse response of a bandpass filter. Obviously, the CWT offers a great degree of freedom in the choice of a wavelet. The inverse transform is defined as

$$f(x) = \left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{W}f(a,b)\psi_{a,b}(x)dadb/a^2\right)/C_\psi.$$

The CWT is highly redundant, and is shift invariant. It is extensively used for the characterization of signals. The evolution of the CWT magnitude across scales provides information about the local regularity of a signal.

2.2. Discrete Wavelet Transform (DWT)

Discrete Wavelet Transform can be derived from the CWT. The discrete wavelet transform (DWT) is in literature commonly associated with signal expansion into (bi-) orthogonal wavelet bases. Thus, as opposed to the highly redundant CWT, there is no redundancy in the DWT of a signal.

Discrete Wavelet Transform can be derived from the CWT. The discrete wavelet transform (DWT) is in literature commonly associated with signal expansion into (bi-) orthogonal wavelet bases. Thus, as opposed to the highly redundant CWT, there is no redundancy in the DWT of a signal; the scale is sampled at dyadic steps $a \in \{2^j : j \in Z\}$, and the position is sampled proportionally to the scale $b \in \{k2^j : (j, k) \in Z^2\}$.

By no means can a DWT be understood as a simple sampling from a CWT. In the first place, the choice of a wavelet is now far more restrictive: if we are dealing with finite-energy signals $f(x) \in L^2(R)$, the wavelet $\psi(x)$ has to be chosen such that $\{\psi(2^{-j}(x - 2^j k))\}$ $(j,k) \in Z^2$ is a basis of $L^2(R)$. The systematic framework for constructing wavelet bases, known as the multiresolution analysis.

The orthogonal wavelets are rarely available as closed form expressions, but rather obtained through a computational procedure which uses discrete filters. The term “wavelets” refers to a orthonormal basis function that is generated by the translation and dilation of scaling function Φ and the mother wavelet ψ. A discrete wavelet transform is a finite scale multi resolution representation of a discrete function. Discrete wavelet transform is orthogonal and invertible where the inverse transform is expressed as the matrix is the transpose of the transform matrix. The wavelet function is localized in space, unlike sines and cosines in Fourier transform. Similar to sines and cosines the individual wavelet functions are localized in frequency. The wavelet basis is defined a

$$\psi_{(j,k)}(x) = 2^{j/2}\psi(2^j x - k)$$

The scaling function is mathematically given by

$$\phi_{(j,k)}(x) = 2^{j/2}\phi(2^j x - k)$$

Where ψ is the wavelet function and j and k are integers that scale and dilate the wavelet function. Factor ‘j’ in Equations is known as the scale index, which represents the width of the wavelet. The location index k gives the position. The wavelet function is dilated by powers of two and is translated by k which is an integer. In terms of the wavelet coefficients, the wavelet equation is

$$\psi(x) = \sum_{k}^{N-1} g_k \sqrt{2\phi(2x-k)}\,.$$

The
function Φ(x) represents a scaling function and the coefficients h0, h1,….. are low pass scaling coefficients. The wavelet and scaling coefficients are related by the a quadrature mirror relationship which is given as

$$g_n = (-1)^n h_{1-n+N}$$

Where N is
the number of vanishing moments.

2.3Properties of DWT

Properties of a discrete wavelet transforms are given below.

- It provides a fast linear operation which can be efficiently applied on data vectors having length as integral power of 2.
- Discrete wavelet transform is invertible and orthogonal. The scaling function Φ and the wavelet function ψ are orthogonal to each other in L2(0, 1), i.e., $< \Phi, \psi> = 0$.
- The wavelet function is localized in terms of space and frequency.
- The coefficients satisfies some constraints

$$\sum_{i=0}^{2N-1} h_i = \sqrt{2}$$

$$\sum_{i=0}^{2N-1} h_i h_{i+2l} = \delta_{1,0}$$

Here δ is the delta function and l is the location index.

$$\sum_{i=0}^{2N-1} (-1)^i i^k h_i = 0$$

3. Karhunen-Loeve Transform (KLT)

Karhunen-Loeve Transform (KLT) which was built on statistical-based properties. The outstanding advantage of KLT is a good de-correlation. In the MSE (Mean Square Error) sense, it is the best transform, and it has an important position in the data compression technology.

KLT has four characteristics:

- *De-correlation:* After transform the weight if vector signal Y unrelated.
- *Energy concentration:* After transform of N-dimensional vector signal, the maximum variance is in the former of M lower sub-vector.
- *Under measuring of the MSE:* The distortion is less than other transform. It is the sum of the sub-vectors which were omitted.
- No quick algorithm and the different signal sample collection has different transformation matrix. (it is the shortcoming of KLT)

KLT is chosen over other transforms as:

- DCT which is short of Discrete Cosine Transform, it is very similar with DFT (Discrete Fourier Transform) but it only uses real number. Both KLT and DCT are used in image processing. For the DCT, especially the DCT-II is always used to lossless data compression for signal and image. It has an "energy concentration" property: most the signal information tends to be concentrated in a few low-frequency components of the DCT, approaching the KLT for signals based on certain limits of Markov process. Then its de-correlation gets close to KLT. So the DCT is almost as good as KLT for a 1^{st} order Markov process.
- There is no fast algorithm in KLT which is a big barrier in practical application. The DCT has a fast algorithm. Then the DCT can achieve much faster compression than the KLT, while the DCT leads to relatively large degradation of compression quality at the same compression ratio compared to the KLT.
- A fast Fourier transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and it's inverse. The FFT has been applied widely, such as digital signal processing, solving partial differential equations to algorithms for quick multiplication of large integers, and so on. Beyond the FFT, the KLT is used to extract of weak signal from noise plus data compression. Both KLT and FFT are used to image processing and signal processing.
- The KLT is a mathematical tool superior to the FFT in that it accuracy applies to any finite bandwidth, rather than applying to infinitely small bandwidth only (i.e. to monochromatic signals) as the FFT does. Also, the KLT applies to both stationary and non-stationary processes, but the FFT works only for stationary input stochastic processes. The KLT is defined for any finite time interval, but the FFT is plagued by the "window" problems. For the KLT it needs high computational burden because of no "fast" KLT. Comparing with FFT, it is the fast algorithm than FFT.
- KLT is built on statistical-based properties. The WT based on waveform transform. They are based on different foundation. The outstanding advantage of KLT is a good de-correlation. For the WT, the basic method is used to denosing and analyze signal.

The KLT and WT are always used in image processing.

4. CONCLUSION

Denoising plays a very important role in the field of the image processing. It is often done before the image data is to be analyzed. Denoising is mainly used to remove the noise that is present and retains the significant information, regardless of the frequency contents of the signal. Denoising has to be performed to recover the useful information. The main purpose of an image-denoising algorithm is to eliminate the unwanted noise level while preserving the important features of an image. The WT shows an excellent performance in the denoising field while KLT shows a good performance in the signal reconstructed ability. WT has been widely used; KLT is not as popular as WT, the reason caused by its different mathematical structures.

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